Please show all work in the spaces provided. If work is on a separate sheet, clearly enumerate and staple to the back of this packet.

ALGEBRAIC MANIPULATION

Solve for the indicated variable. Express all answers as fractions in lowest terms.
To evaluate, substitute the given value(s) of the variable(s) and use order of operations to find the value of the resulting numerical expression.

1. Solve for x.

$$2[x + 3(x - 1)] = 18$$

$2[x+3x-3]=18$
$2(4x-3)=18$
$8x-6=18$
$8x=24$
$\boxed{x=3}$

2. Solve for x.

$$-(1 + 7x) - 6(-7 - x) = 36$$

$-1-7x+42+6x=36$
$41-7x+6x=36$
$41-x=36$
$-x=-5$
$\boxed{x=5}$

3. Solve for x.

$$\frac{x-2}{3} = \frac{2x+1}{4}$$

$4(x-2)=3(2x+1)$
$4x-8=6x+3$
$-2x-8=3$
$-2x=11$
$\boxed{x=-\frac{11}{2}}$

4. Solve for x.

$$6 + 2x(x - 3) = 2x^2$$

$6+2x^2-6x=2x^2$
$6-6x=0$
$6=6x$
$1=x$
$\boxed{x=1}$

5. Solve for x.

$$2x^2 = 50$$

$x^2=25$
$x=\pm\sqrt{25}$
$\boxed{x=\pm5}$

6. Solve for x.

$$|x + 4| + 8 = 2x + 4$$

$|x+4|=2x-4$

$x+4=2x-4$
$-x+4=-4$
$-x=-8$
$\boxed{x=8}$

~~$x+4=-(2x-4)$~~
~~$x+4=-2x+4$~~
~~$-3x+4=4$~~
~~$-3x=0$~~
~~$x=0$~~

7. The relationship between the sale price S, the list price L, and the discount rate r is given by $S = L - rL$. Solve for r.

$S=L-rL$
$S-L=-rL$
$\boxed{\frac{S-L}{-L}=r}$

8. Solve for m.

$$g = 4cm - 3m$$

$g=m(4c-3)$
$\boxed{\frac{g}{4c-3}=m}$

9. Evaluate $\frac{-b+\sqrt{b^2-4ac}}{2a}$ if $a = 1,\ b = -4,\ c = -21$

$$\frac{-(-4)+\sqrt{(-4)^2-4(1)(-21)}}{2(1)}$$

$$= \frac{4+\sqrt{16+84}}{2}$$

$$= \frac{4+\sqrt{100}}{2} = \frac{4+10}{2} = \frac{14}{2} = \boxed{7}$$

10. Evaluate $A = P\left(1+\frac{r}{n}\right)^{nt}$ if $P = 650,\ r = 6\%,\ n = 2,\ t = 15$

$$A = 650\left(1+\frac{.06}{2}\right)^{2(15)}$$

$$A = 650(1.03)^{30}$$

$$A \approx 1577.720606$$

$$\boxed{A \approx 1577.72}$$

OPERATIONS WITH POLYNOMIALS

Perform the indicated operations and simplify completely (meaning collect similar terms).

11. $(-x)(-3y)(-5z)$

$$= (3xy)(-5z)$$

$$= \boxed{-15xyz}$$

12. $(4n-3)^2$

$$= (4n-3)(4n-3)$$

$$= 16n^2 - 12n - 12n + 9$$

$$= \boxed{16n^2 - 24n + 9}$$

13. $(7x-3)(7x+3)$

$$= 49x^2 + 21x - 21x - 9$$

$$= \boxed{49x^2 - 9}$$

14. $(5x^2-4) - 2(3x^2+8x+4)$

$$= 5x^2 - 4 - 6x^2 - 16x - 8$$

$$= -x^2 - 16x - 4 - 8$$

$$= \boxed{-x^2 - 16x - 12}$$

15. $(x^2+x-3)(3x^2-x+3)$

$$= 3x^4 - x^3 + 3x^2 + 3x^3 - x^2 + 3x - 9x^2 + 3x - 9$$

$$= 3x^4 + 2x^3 + 3x^2 - x^2 - 9x^2 + 3x + 3x - 9$$

$$= 3x^4 + 2x^3 - 7x^2 + 3x + 3x - 9$$

$$= \boxed{3x^4 + 2x^3 - 7x^2 + 6x - 9}$$

16. $(n^2-4n-6) + (-3n^2+2n-9)$

$$= n^2 - 4n - 6 - 3n^2 + 2n - 9$$

$$= -2n^2 - 4n + 2n - 6 - 9$$

$$= -2n^2 - 2n - 6 - 9$$

$$= \boxed{-2n^2 - 2n - 15}$$

Properties of Exponents

Let a and b be real numbers and let m and n be rational numbers, such that the quantities in each property are real numbers.

Property Name	Definition	Example
Product of Powers	$a^m \cdot a^n = a^{m+n}$	$2^3 \cdot 2^2 = 2^{(3+2)} = 2^5 = 32$
Power of a Power	$(a^m)^n = a^{mn}$	$(x^4)^2 = x^{(4 \cdot 2)} = x^8$
Power of a Product	$(ab)^m = a^m b^m$	$(2x)^3 = 2^3 \cdot x^3 = 8 \cdot x^3 = 8x^3$
Negative Exponent	$a^{-m} = \frac{1}{a^m}$, *where* $a \neq 0$	$x^{-2} = \frac{1}{x^2}$
Zero Exponent	$a^0 = 1$, *where* $a \neq 0$	$213^0 = 1$
Quotient of Powers	$\frac{a^m}{a^n} = a^{m-n}$, *where* $a \neq 0$	$\frac{4^7}{4^5} = 4^{(7-5)} = 4^2 = 16$
Power of a Quotient	$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}, b \neq 0$	$\left(\frac{2}{3}\right)^3 = \frac{2^3}{3^3} = \frac{8}{27}$

OPERATIONS WITH EXPONENTS

Simplify each expression using the exponent rules. Answers should be written using positive exponents.

17. $\frac{y^{12}}{y^8}$ $= y^{12-8} = \boxed{y^4}$

18. $(-4a^5b^0c)^2$

$= (-4)^2 a^{5(2)} b^{0(2)} c^{1(2)}$

$= 16a^{10}b^0c^2$

$= \boxed{16a^{10}c^2}$

19. $(2x^4)^{-3}$

$= \frac{1}{(2x^4)^3}$

$= \frac{1}{2^3 x^{4(3)}}$

$= \boxed{\frac{1}{8x^{12}}}$

20. $(x^{2y})(2x^y)(x^{y+3})$

$= 2 \cdot x^{2y+y+y+3}$

$= \boxed{2x^{4y+3}}$

21. $\left(\frac{3}{x-3}\right)^7 = \frac{3^7}{(x-3)^7}$

$= \boxed{\frac{2187}{(x-3)^7}}$

22. $\frac{5x^3y^9}{30x^4y^{-2}} = \frac{1x^3y^9}{6x^4y^{-2}}$

$= \frac{x^3y^9y^2}{6x^4}$

$= \frac{x^3y^{11}}{6x^4} = \boxed{\frac{y^{11}}{6x}}$

SYSTEMS OF EQUATIONS

Solve each system of equations algebraically (using the substitution or elimination method)

23. Solve the system of equations by substitution.

$y = 2x + 4$

$-3x + y = -9$

$-3x + (2x+4) = -9$

$-3x + 2x + 4 = -9$

$-x + 4 = -9$

$-x = -13$

$x = 13$

$y = 2(13) + 4$

$y = 26 + 4$

$y = 30$

$\boxed{(13, 30)}$

24. Solve the system of equations by elimination.

$2x + 3y = 6$

$-3x + 2y = 17$

$2x + 3y = 6 \xrightarrow{\times 3} 6x + 9y = 18$

$-3x + 2y = 17 \xrightarrow{\times 2} -6x + 4y = 34$

$13y = 52$

$y = 4$

$2x + 3(4) = 6$

$2x + 12 = 6$

$2x = -6$

$x = -3$

$\boxed{(-3, 4)}$

FACTORING POLYOMIALS

Factor the following completely.

25. $16y^2 + 8y$

GCF

$\boxed{8y(2y+1)}$

26. $x^2 - 1$

Difference of Two Perfect Squares

$\boxed{(x+1)(x-1)}$

27. $a^2 - 6a - 40$

$\boxed{(a-10)(a+4)}$

40

1x40

2x20

4x10

5x8

Trinomial where a=1

28. $49x^2 - 100y^2$

Difference of Two Perfect Squares

$\boxed{(7x-10y)(7x+10y)}$

29. $6x^2 + 13x - 5$

$a \cdot c = -30$

$b = 13$

30
1×30
2×15
3×10
5×6

$6x^2 - 2x + 15x - 5$

$2x(3x-1) + 5(3x-1)$

$\boxed{(3x-1)(2x+5)}$

Trinomial where $a > 1$ → Factor by Grouping

30. $3x^3 + x^2 - 15x - 5$

$x^2(3x+1) - 5(3x+1)$

$\boxed{(3x+1)(x^2-5)}$

Factor by Grouping

INEQUALITIES

Solve and graph the following inequalities on a number line. Write your final answer in interval notation.

31. $4(x+5) > 2x - 8$

$4x + 20 > 2x - 8$

$4x > 2x - 28$

$2x > -28$

$\boxed{x > -14}$

$\boxed{(-14, \infty)}$

32. $2x - 3 > 9$ *or* $3x \geq 12$

$2x > 12$

$x > 6$ OR $x \geq 4$

$\boxed{[4, \infty)}$

33. $-3 \leq 2x - 11 < 7$

$8 \leq 2x < 18$

$\boxed{4 \leq x < 9}$

$\boxed{[4, 9)}$

34. SOLVE BY COMPLETING THE SQUARE

$x^2 + 10x - 25 = 0$

$x^2 + 10x = 25$

$c = \left(\frac{b}{2}\right)^2$ $\left(\frac{10}{2}\right)^2 = (5)^2 = 25$

$x^2 + 10x + 25 = 25 + 25$

$(x+5)^2 = 50$

$x + 5 = \pm\sqrt{50}$

$x + 5 = \pm 5\sqrt{2}$

$\boxed{x = -5 \pm 5\sqrt{2}}$

35. SOLVE USING THE QUADRATIC FORMULA

$2x^2 - 14x + 40 = 3x^2 - 16x + 32$

$0 = x^2 - 2x - 8$

$a = 1$
$b = -2$
$c = -8$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-8)}}{2(1)}$

$x = \frac{2 \pm \sqrt{4 + 32}}{2}$

$x = \frac{2 \pm \sqrt{36}}{2}$

$x = \frac{2 \pm 6}{2}$

$x = \frac{2+6}{2} = \frac{8}{2}$

$\boxed{x = 4}$

$x = \frac{2-6}{2} = \frac{-4}{2}$

$\boxed{x = -2}$

SIMPLIFYING RADICALS

An expression under a radical sign is in simplest radical form when:

- *there is no integer under the radical sign with a perfect square factor,*
- *there are no fractions under the radical sign,*
- *there are no radicals in the denominator*

36. $\sqrt{50}$

$\sqrt{50} = \sqrt{2 \cdot 25}$

$= \sqrt{2} \cdot \sqrt{25}$

$= \sqrt{2} \cdot 5$

$= 5\sqrt{2}$

$\sqrt{2 \cdot 5 \cdot 5}$

$5\sqrt{2}$

37. $(3 + 2\sqrt{3})(2 - 5\sqrt{3})$

$= 6 - 15\sqrt{3} + 4\sqrt{3} - 10\sqrt{9}$

$= 6 - 11\sqrt{3} - 10\sqrt{9}$

$= 6 - 11\sqrt{3} - 10(3)$

$= 6 - 11\sqrt{3} - 30$

$= -24 - 11\sqrt{3}$

38. $3\sqrt{2} + 4\sqrt{8} - 5\sqrt{50}$

$4(2)\sqrt{2} \quad (5)5\sqrt{2}$

$= 3\sqrt{2} + 8\sqrt{2} - 25\sqrt{2}$

$= 11\sqrt{2} - 25\sqrt{2}$

$= -14\sqrt{2}$

39. $\sqrt{\frac{6}{27}}$

$= \frac{\sqrt{6}}{\sqrt{27}} = \frac{\sqrt{6}}{3\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$

$= \frac{\sqrt{18}}{3(3)}$

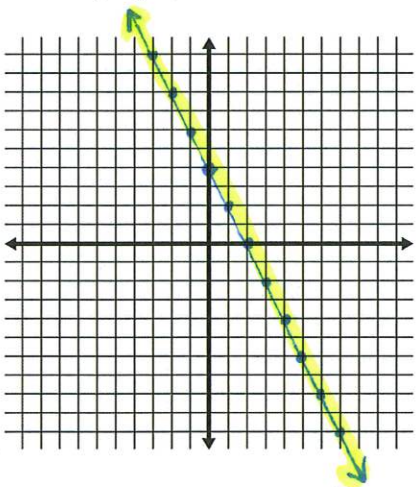
$= \frac{\sqrt{18}}{9}$

$= \frac{3\sqrt{2}}{9}$

$= \frac{\sqrt{2}}{3}$

LINES AND COORDINATE GEOMETRY

40. Graph $y = -2x + 4$

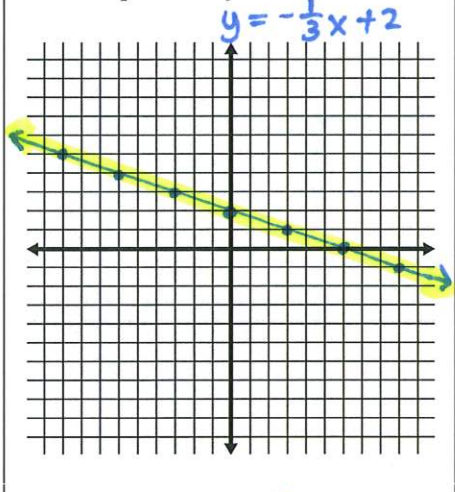


Slope = m = $\frac{-2}{1}$

y-intercept = b = $(0, 4)$

41. Graph $x + 3y = 6$

$y = -\frac{1}{3}x + 2$



Slope = m = $-\frac{1}{3}$

y-intercept = b = $(0, 2)$

42. Determine whether the lines are parallel, perpendicular, or neither. Explain your reasoning.

Eq 1 $2x + 3y = 12$

Eq 2 $3x + 2y = 24$

Eq 1 $2x + 3y = 12$

$3y = -2x + 12$

$y = -\frac{2}{3}x + 4$

$m = -\frac{2}{3}$

Eq 2 $3x + 2y = 24$

$2y = -3x + 24$

$y = -\frac{3}{2}x + 12$

$m = -\frac{3}{2}$

Lines are neither parallel nor perpendicular. Lines are intersecting.

43. In 1998, Matt had **$429** in his bank account. By 2010, he had **$2540**. Find the rate of change for his bank account.

(1998, 429)

(2010, 2540)

$\frac{y_2-y_1}{x_2-x_1} = \frac{2540-429}{2010-1998} = \frac{2111}{12}$

DOMAIN AND RANGE

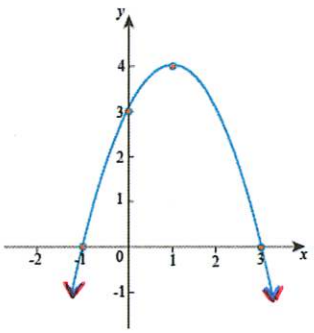
All of the values that can go into a relation or function (input) are called the domain.
All of the values that come out of a relation or function (output) are called the range.

44. $(1,2), (-3,8), (-9,6), \left(\frac{1}{2}, 5\right)$

Domain: $\{-9, -3, \frac{1}{2}, 1\}$

Range: $\{2, 5, 6, 8\}$

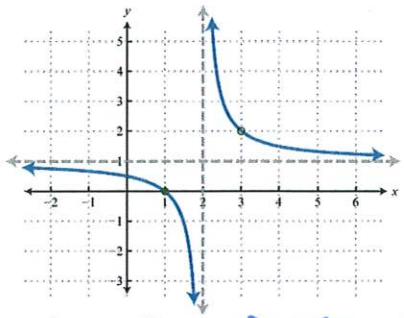
45.



Domain: $(-\infty, \infty)$

Range: $(-\infty, 4]$

46.



Domain: $(-\infty, 2) \cup (2, \infty)$

Range: $(-\infty, 1) \cup (1, \infty)$

EVALUATING FUNCTIONS

To evaluate, substitute the given value(s) of the variable(s) and use order of operations to find the value of the resulting numerical expression.

47. If $f(x) = -2x^2 + x + 3$, evaluate each of the following:

a) $f(-2) = -2(-2)^2 + (-2) + 3$

$f(-2) = -7$

b) $f(3m) = -2(3m)^2 + (3m) + 3$

$f(3m) = -18m^2 + 3m + 3$

c) $f(p^5) = -2(p^5)^2 + (p^5) + 3$

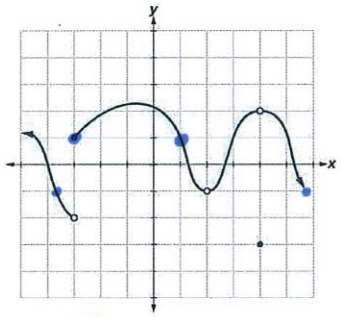
$f(p^5) = -2p^{10} + p^5 + 3$

d) $f(x+h) = -2(x+h)^2 + (x+h) + 3$

$f(x+h) = -2(x^2 + 2xh + h^2) + x + h + 3$

$f(x+h) = -2x^2 - 4xh - 2h^2 + x + h + 3$

48. The graph of a function g is given.



a) $g(-3) = 1$

b) $g(1) = 1$

c) Find the values of x for which is $g(x) = -1$

$x \approx 3.5$

$x \approx 5.5$

49. Evaluate the following functions given:

$$f(x) = 4x - 12x^2 + 7x^3 \qquad g(x) = 2\left(\frac{1}{3}\right)^x \qquad h(x) = \frac{4-x}{-x^2}$$

a) $g(2) = 2\left(\frac{1}{3}\right)^2$

$= 2\left(\frac{1}{3}\right)\left(\frac{1}{3}\right)$

$= 2\left(\frac{1}{9}\right)$

$= \boxed{\frac{2}{9}}$

b) $h(f(1)) = h[4(1) - 12(1)^2 + 7(1)^3]$

$h[4 - 12 + 7]$

$h(-1)$

$= \frac{4-(-1)}{-(-1)^2} = \frac{4+1}{-(1)} = \frac{5}{-1} = \boxed{-5}$

RIGHT TRIANGLE TRIGONOMETRY

Use the Pythagorean Theorem and SOHCAHTOA to help you solve for an unknown side/angle.

50. Find the missing side. Round answer to the nearest tenth.

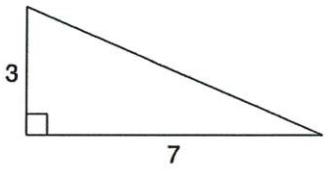
$a^2 + b^2 = c^2$

$(3)^2 + (7)^2 = c^2$

$9 + 49 = c^2$

$58 = c^2$

$\pm\sqrt{58} = c$ → $c = \sqrt{58}$



$\boxed{c \approx 7.6}$

51. Find the missing side. Answer must be in simplest radical form.

$a^2 + b^2 = c^2$

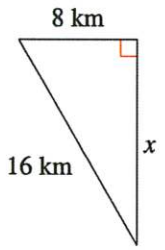
$(8)^2 + b^2 = (16)^2$

$64 + b^2 = 256$

$b^2 = 192$

$b = \sqrt{192}$

$\boxed{b = 8\sqrt{3}\text{ km}}$



52. Find the missing side in exact form.

$\boxed{y = 20}$

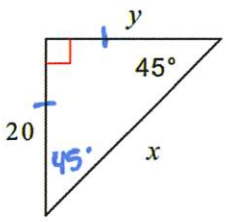
$\boxed{x = 20\sqrt{2}}$

√: $(20)^2 + (20)^2 = x^2$

$400 + 400 = x^2$

$800 = x^2$

$20\sqrt{2} = x$ ✓



isosceles right △

45-45-90° △

53. Find the missing side in exact form.

$\boxed{b = 6}$

$\boxed{a = 12}$

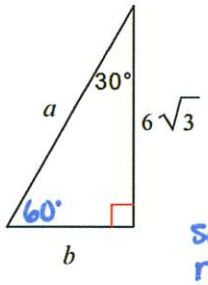
√: $(6)^2 + (6\sqrt{3})^2 = a^2$

$36 + 108 = a^2$

$144 = a^2$

$\sqrt{144} = a$

$12 = a$ ✓



scalene right △

30°-60°-90° △

54. Find the missing side in exact form.

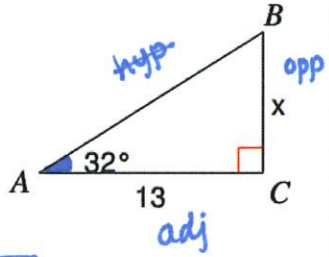
SOH CAH TOA

$\tan(32) = \frac{x}{13}$

$13 \cdot \tan(32) = x$

$\boxed{x = 13 \cdot \tan(32)}$

$x \approx 8.1$



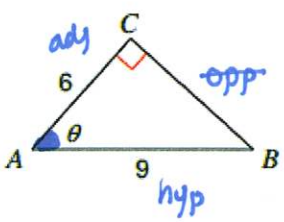
55. Find the missing angle. Round to the nearest tenth.

SOH CAH TOA

$\cos\theta = \frac{6}{9}$

$\theta = \cos^{-1}\left(\frac{6}{9}\right)$

$\boxed{\theta \approx 48.2°}$



#54 + #55 → Be sure calculator is in DEGREES!